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A Study of Families of Bistar and Corona Product of Graph: Reverse Topological Indices

Gowtham, K. J.[1] and Husin, M. N.*[2]

[1]*Department of Mathematics, University College of Science, Tumkur University,*
Tumakuru, Karnataka State, India
[2]*Special Interest Group on Modeling and Data Analytics,*
Faculty of Ocean Engineering Technology and Informatics,
Universiti Malaysia Terengganu, Kuala Nerus 20130, Terengganu

E-mail: nazri.husin@umt.edu.my
**Corresponding author*

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Abstract

In the field of cheminformatics, the amalgamation of graph theory, chemistry, along with technology facilitates the establishment of connections between the structural as well as physiochemical attributes of organic compounds by employing certain valuable graph invariants including the corresponding molecular graph. In this work, we examine reverse topological indices, for instance, the reverse Zagreb index, the reverse arithmetic-geometric, the geometric-arithmetic, the reverse Nirmala indices for the bistar graphs $B(n;m)$, the reverse sum-connectivity index, the reverse Sombor as well as the corona product of $K_m \circ K'_n$.

Keywords: reverse topological indices; bistar graph; the corona product; complete graph.

1 Introduction

In computer networks and other subjects such as coding theory, database management systems, circuit design, secret-sharing schemes, and theoretical chemistry, graph theory is a powerful research tool. Cheminformatics combines chemistry, technology, and graph theory. Utilizing some practical graph invariants and the corresponding molecular graph links the structure and physiochemical characteristics of organic substances. A molecular graph comprises collections of points and lines representing the atoms and covalent bonds that make up a molecule. Due to its useful applications in quantitative structural activity and quantitative structure property research (QSAR/QSPR), the theoretical study of underlying chemical structures using useful graph invariants is a fascinating area of research in mathematical chemistry [26]. The physicochemical characteristics of the chemical compounds are predicted using topological indices. The topological index is used to characterize some molecular graphs property, which are used in theoretical chemistry. The mathematical aspects of modern organic chemistry are addressed in [10] and various molecular descriptor representations and their corresponding molecular descriptors are studied in [22]. The research encompasses a broad spectrum of mathematical and chemical investigations.

In [2], combinatorial aspects are explored, with a focus on computing the Sombor index, average Sombor index, and the reduced Sombor index for the line graph of silicate carbides. In [4] delves into the study of the ratio between geometric and arithmetic means, while [3, 6] introduces the M-polynomial, a tool for efficiently calculating topological invariants of molecular graphs. Furthermore, in [11] investigates the generalization of various graph indices, including the edge version of the Randic, GA, ABC, multiplicative ABC, Zagreb, and inverse sum indices for specific graph networks using the concept of line graphs. In [12], the numerical simulation of the atom-bond connectivity index is explored in the context of QSAR and QSPR studies. In [13], the characterizing the metal-organic framework of iron(III) tetra-p-tolyl porphyrin (FeTPyP) and the CoBHT (CO) lattice. In [19], the investigated two formulas for the number of spanning trees in a chain of diphenylene planar graphs that have connected intersection of one edge with the same sizes. These diverse research efforts contribute to a better understanding of mathematical and chemical aspects in various contexts.

Let $G = (V, E)$ be a finite, undirected and simple graph with vertex set $V(G)$ and edge set $E(G)$. The maximum degree of vertex among the set $V(G)$ is denoted by $\Delta(G)$ and the degree of the vertex v is denoted by d_v. The reverse vertex degree rv of the vertex v is defined as $rv = \Delta(G) - d_v + 1$ [5]. In [7], explores the concept of reverse Laplacian energy and the maximum reverse degree energy of a graph [8]. Furthermore, the research delves into the examination of reversed degree-based topological indices for various materials, such as graphene [17], Vanadium Carbide [23], Ceria Oxide [25], and carbon nanotube [24], metal-organic framework [20], Titania Nanotubes [18], fullerene cage networks [1] and fullerene networks [26]. These studies contribute to a deeper understanding of the topological properties and characteristics of different materials and compounds.

The primary distinction between vertex-degree-based and reverse-vertex-degree-based topological indices lies in the consideration of direct degrees versus their reciprocals. Reverse-vertex-degree-based indices offer a unique perspective and are particularly useful in diverse fields, including cheminformatics, ecology, network analysis, and communication networks [16]. They provide insights into the relative importance of vertices in a network from a different angle, which can lead to novel applications and insights. Throughout this work, u v refers to the adjacent vertices u as well as v in the graph G. Let us first go over some of the common definitions before moving on to our main results. The applications of reverse topological indices are discussed in [9]. Throughout this work, uv denotes the adjacent vertices u and v in the graph G. Let us first go over some of the common definitions before moving on to our main results.

Definition 1.1. [5] *The first and second reverse Zagrab indices are defined as:*

$$RM_1(G) = \sum_{u} r_u^2, \tag{1}$$

$$RM_2(G) = \sum_{u\sim v} (r_u \cdot r_v). \tag{2}$$

Definition 1.2. [15] *The reverse sum-Connectivity index of a graph G is defined as:*

$$RSCI(G) = \sum_{u\sim v} \frac{1}{\sqrt{r_u + r_v}}. \tag{3}$$

Definition 1.3. [14, 16] *The reverse arithmetic-geometric, and geometric-arithmetic indices of a graph G is defined as:*

$$RAG(G) = \sum_{u\sim v} \frac{r_u + r_v}{2\sqrt{r_u \cdot r_v}}, \tag{4}$$

$$RGA(G) = \sum_{u\sim v} \frac{2\sqrt{r_u \cdot r_v}}{r_u + r_v}. \tag{5}$$

Definition 1.4. [15, 21] *The reverse Sombor index of a graph G is defined as:*

$$RSO(G) = \sum_{u\sim v} \sqrt{r_u^2 + r_v^2}. \tag{6}$$

Definition 1.5. [9] *The reverse Nirmala index of a graph G is defined as:*

$$RN(G) = \sum_{u\sim v} \sqrt{r_u + r_v}. \tag{7}$$

2 Reverse Topological indices of Families of Bistar Graph $[G = B(n; m)]$

The bistar graph, denoted by $B(n; m)$, is a graph created by connecting the centers of two-star graphs of orders m and n, i.e., $K_{1,m}$ and $K_{1,n}$, by an edge. The values of families of bistar graphs are computed using topological indices that we present in this section. Figure 1 depicts a representation of the bistar graph $B(n, m)$ of order m as well as n.

Theorem 2.1. *The reverse sum-connectivity index with respect to bistar graph $B(n; m)$ families is*

$$RSCI(G) = \begin{cases} \dfrac{n}{\sqrt{n+2}} + \dfrac{1}{\sqrt{n-m+2}} + \dfrac{m}{\sqrt{2n-m+2}}, & \text{if } n > m, \\[2ex] \dfrac{n}{\sqrt{2m-n+2}} + \dfrac{1}{\sqrt{m-n+2}} + \dfrac{m}{\sqrt{m+2}}, & \text{if } m > n, \\[2ex] \dfrac{2n+1}{\sqrt{n+2}}, & \text{if } n = m. \end{cases}$$

Proof. The three cases given below were considered:

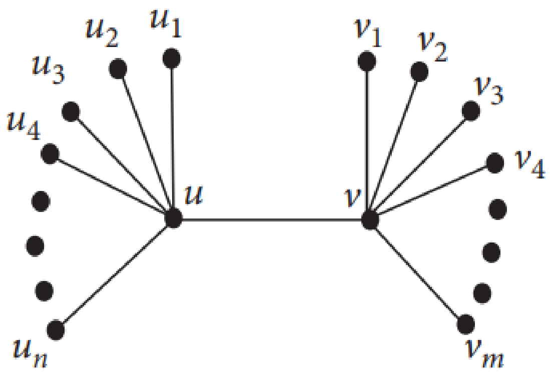

Figure 1: A representation of bistar graph $B(n;m)$ of order m and n.

Case(i) Let $n > m$. Select a vertex u in $B(n;m)$ of degree $(n+1)$. The maximum degree among the vertices of G is $n+1$ (i.e., $\Delta(G) = n+1$). Then the reverse vertex degree of u is given by $r_u = \Delta(G) - d_u + 1 = n+1-(n+1)+1 = 1$. There are n vertices $u_1, u_2, u_3, \ldots, u_n$ of degree 1 and their reverse degree of $u_i's$ will be $r_{u_i} = n+1$. Now, the reverse sum-connectivity index for the vertices u and $u_i's$ (where $i = 1,2,3,\ldots,n$) is given by

$$\sum_{u\sim u_i} \frac{1}{\sqrt{r_u + r_{u_i}}} = \frac{n}{\sqrt{n+2}}. \tag{8}$$

The other vertex which is adjacent to u is v of reverse vertex degree $r_v = n-m+1$. For the vertices u, and v, we have

$$\frac{1}{\sqrt{r_u + r_v}} = \frac{1}{\sqrt{n-m+1}}. \tag{9}$$

Next, we select a vertex v of degree $m+1$ on $B(n;m)$. Now, there are m vertices $v_1, v_2, v_3, \ldots, v_m$ of reverse vertex degree $r_{v_j} = n+1$. For the vertices v and $v_j's$, the sum is obtained as follows:

$$\sum_{v\sim v_j} \frac{1}{\sqrt{r_v + r_{v_j}}} = \frac{m}{\sqrt{2n-m+2}}. \tag{10}$$

For $n > m$, the sum-connectivity index of $B(n;m)$ is obtain by adding equations (8) to (10).

Case(ii) Suppose $m > n$. In this case, the maximum degree among the vertices of G is $m+1$, the reverse vertex degree of u is $r_u = m-n+1$, and reverse vertex degree of $u_i's$ is $r_{u_i} = m+1$ So,

$$\sum_{u\sim u_i} \frac{1}{\sqrt{r_u + r_{u_i}}} = \frac{n}{\sqrt{2m-n+2}}. \tag{11}$$

The reverse degree of v is $r_v = 1$ and the reverse vertex of each $v_j's$ is $r_{v_j} = m+1$. So,

$$\frac{1}{\sqrt{r_u + r_v}} = \frac{1}{\sqrt{m-n+2}}. \tag{12}$$

And

$$\sum_{v \sim v_j} \frac{1}{\sqrt{r_u + r_{v_j}}} = \frac{m}{\sqrt{m+2}}. \tag{13}$$

For $m > n$, the sum-connectivity index of $B(n; m)$ is obtain by adding equations (11) to (13).

Case(iii) Suppose $m = n$. Here, the reverse vertex degree of u and v is $r_u = 1 = r_v$. The reverse vertex degree of each $u_i's$ is $r_{u_i} = n + 1$ and each $v_j's$ is $r_{v_j} = n + 1$. We have,

$$\sum_{u \sim u_i} \frac{1}{\sqrt{r_u + r_{u_i}}} = \frac{n}{\sqrt{n+2}}, \tag{14}$$

$$\frac{1}{\sqrt{r_u + r_v}} = \frac{1}{\sqrt{n+2}}, \tag{15}$$

and

$$\sum_{v \sim v_j} \frac{1}{\sqrt{r_v + r_{v_j}}} = \frac{n}{\sqrt{n+2}}. \tag{16}$$

For $m = n$, the sum-connectivity index of $B(n; m)$ is obtain by adding equations (14) to (16).

□

Theorem 2.2. *The reverse Zagreb indices of the families of bistar graphs $B(n; m)$ are*

$$RM_1(G) = \begin{cases} (n+m)(n+1)^2 + (n-m+1)^2 + 1, & \text{if } n > m, \\ (n+m)(m+1)^2 + (m-n+1)^2 + 1, & \text{if } m > n, \\ 2n(n+1)^2 + 2, & \text{if } n = m. \end{cases}$$

$$RM_2(G) = \begin{cases} n(n+1) + (n-m+1)[mn+m+1], & \text{if } n > m, \\ n(m+1) + (m-n+1)[mn+n+1], & \text{if } m > n, \\ 2n(n+1) + 1, & \text{if } n = m. \end{cases}$$

Proof. The following three cases are considered:

Case(i) Let $n > m$. We select a vertex u on $B(n; m)$ which as maximum vertex degree $n+1$ and its reverse vertex degree of u is $r_u = 1$. So,

$$r_u^2 = 1, \tag{17}$$

Since there are n vertices $u_1, u_2, u_3, \ldots, u_n$ of $B(n; m)$ of reverse vertex degree $r_{u_i} = n+1$ which are adjacent to u. Thus we have,

$$\sum_{u_i} r_{u_i}^2 = n(n+1)^2. \tag{18}$$

Provided that the reverse vertex degree of u is $r_u = 1$, the other vertex which is adjacent to u is v of reverse vertex degree $r_v = n - m + 1$. For the vertex v, we have

$$r_v^2 = (n - m + 1)^2. \tag{19}$$

Provided that the reverse vertex degree of v is $r_v = n - m + 1$, the other vertices which is adjacent to v are m in number of reverse vertex degree $r_{v_j} = n + 1$,

$$\sum_{v_j} r_{v_j}^2 = m(n + 1)^2. \tag{20}$$

For $n > m$, $RM_1(G)$ is obtained by adding equations (17) to (20).

Case(ii) Suppose $m > n$. Here the reverse vertex degree of the vertex u, v, $u_i's$, and $v_j's$ are given by, $r_u = m - n + 1$, $r_{u_i} = m + 1$, $r_v = 1$, and $r_{v_j} = m + 1$ respectively. So, we have

$$r_u^2 = (m - n + 1)^2, \tag{21}$$

$$\sum_{u_i} r_{u_i}^2 = n(m + 1)^2, \tag{22}$$

$$r_v^2 = 1, \tag{23}$$

$$\sum_{v_j} r_{v_j}^2 = m(m + 1)^2. \tag{24}$$

For $m > n$, $RM_1(G)$ is obtained by adding equations (21) to (24).

Case(iii) Suppose $m = n$. Here the reverse vertex degree of the vertex u, v, $u_i's$, and $v_j's$ are given by, $r_u = 1$, $r_{u_i} = n + 1$, $r_v = 1$, and $r_{v_j} = n + 1$ respectively. So, we have

$$r_u^2 = 1, \tag{25}$$

$$\sum_{u_i} r_{u_i}^2 = n(n + 1)^2, \tag{26}$$

$$r_v^2 = 1, \tag{27}$$

$$\sum_{v_j} r_{v_j}^2 = n(n + 1)^2. \tag{28}$$

For $m = n$, $RM_1(G)$ is obtained by adding equations (25) to (28).

Similarly, one can easily prove the same for $RM_2(G)$.

□

Theorem 2.3. *The reverse arithmetic-geometric and geometric-arithmetic indices of the families of bistar graphs $B(n;m)$ are*

$$RAG(G)=\begin{cases}\dfrac{n(n+2)}{2\sqrt{n+1}}+\dfrac{n-m+2}{2\sqrt{n-m+1}}+\dfrac{m(2n-m+2)}{2\sqrt{(n+1)(n-m+1)}}, & \text{if } n>m,\\[2ex] \dfrac{n(2m-n+2)}{2\sqrt{(n+1)(n-m+1)}}+\dfrac{m-n+2}{2\sqrt{m-n+1}}+\dfrac{m(m+2)}{2\sqrt{m+1}}, & \text{if } m>n,\\[2ex] \dfrac{n(n+2)}{\sqrt{n+1}}+1, & \text{if } n=m.\end{cases}$$

$$RGA(G)=\begin{cases}\dfrac{2n\sqrt{n+1}}{n+2}+\dfrac{2\sqrt{n-m+1}}{n-m+1}+\dfrac{2m\sqrt{(n+1)(n-m+1)}}{2n-m+2}, & \text{if } n>m,\\[2ex] \dfrac{2n\sqrt{(m+1)(m-n+1)}}{2m-n+2}+\dfrac{2\sqrt{m-n+1}}{m-n+2}+\dfrac{2m\sqrt{m+1}}{m+2}, & \text{if } m>n,\\[2ex] \dfrac{4n\sqrt{n+1}}{n+2}+1, & \text{if } n=m.\end{cases}$$

Proof. Theorem 2.1 is analogous to this proof. ☐

Theorem 2.4. *The reverse Sombor index of families of bistar graph $G=B(n;m)$ is*

$$RSO(G)=\begin{cases}n\sqrt{(n+1)^2+1}+\sqrt{(n-m+1)^2+1}+m\sqrt{(n-m+1)^2+(n+1)^2}, & \text{if } n>m,\\[1ex] n\sqrt{(m-n+1)^2+(m+1)^2}+\sqrt{(m-n+1)^2+1}+m\sqrt{(m+1)^2+1}, & \text{if } n>m,\\[1ex] (2n+2)\sqrt{1+(n+1)^2}, & \text{if } m=n.\end{cases}$$

Proof. Theorem 2.1 is analogous to this proof. ☐

Theorem 2.5. *The reverse Nirmala index of families of bistar graph $B(n;m)$ is*

$$RN(G)=\begin{cases}n\sqrt{n+2}+\sqrt{n-m+2}+m\sqrt{2n=m+2}, & \text{if } n>m,\\[1ex] n\sqrt{2m-n+2}+\sqrt{m-n+2}+m\sqrt{m+2}, & \text{if } n>m,\\[1ex] (2n+2)\sqrt{n+2}, & \text{if } m=n.\end{cases}$$

Proof. Theorem 2.1 is analogous to this proof. ☐

2.1 Reverse topological indices of families of corona product of graph $[G=K_m\circ K'_n]$

In this section, by considering the corona product with respect to the complement of K_n of order n as well as the complete graph K_m having order m, we discover the reverse topological indices of the corona product families with respect to graph $K_m\circ K'_n$. And a representation of corona product $K_3\circ K'_4$ is shown in Figure 2.

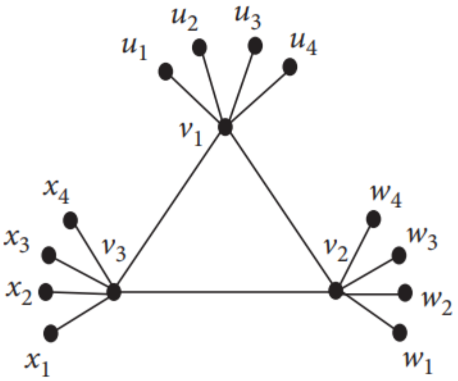

Figure 2: A representation of corona product $K_3 \circ K_4'$.

Theorem 2.6. *The reverse sum-connectivity index of families of corona product of graph $K_m \circ K_n'$ is*

$$RSCI(G) = \frac{mn}{\sqrt{m+n}} + \frac{m(m-1)}{2\sqrt{2}}.$$

Proof. Initially, a vertex v_1 on $K_m \circ K_n'$ as v_1 is adjacent to $m-1$ vertices $v_2, v_3, \ldots, v_m$ and n vertices $u_1, u_2, \ldots, u_n$. Thus, the degree of v_1 is $m+n-1$ and it is the greatest degree among the vertices of $K_m \circ K_n'$ (i.e. $\Delta(G) = m+n-1$). The reverse vertex degree of v_1 is $r_{v_1} = 1$ and reverse vertex degree of each $u_i' s$ is $r_{u_i} = m+n-1$, where $(i = 1, 2, \ldots, n)$. Moreover, the their sum is given below

$$\sum_{v_1 \sim u_i} \frac{1}{\sqrt{r_{v_1} + r_{u_i}}} = \frac{n}{\sqrt{n+m}}. \tag{29}$$

Provided that it is a symmetric, similar finding is gained for the remaining $(m-1)$ vertices $v_2, v_3, \ldots, v_m$. By combining all the findings for all $v_j' s$ then equation (29) becomes,

$$\sum_{v_1 \sim u_i} \frac{1}{\sqrt{r_{v_1} + r_{u_i}}} + \sum_{v_2 \sim w_i} \frac{1}{\sqrt{r_{v_1} + r_{u_i}}} + \cdots + \sum_{v_m \sim x_i} \frac{1}{\sqrt{r_{v_m} + r_{x_i}}} = \frac{mn}{\sqrt{m+n}}. \tag{30}$$

Since v_1 is also adjacent to $v_2, v_3, \ldots, v_m$ of reverse vertex degree 1. Therefore, the sum of vertex v_1 and v_j(where $j = 2, 3, \ldots, m$) with $m-1$ vertices, then we obtained,

$$\begin{aligned}\sum_{v_1 \sim v_j} \frac{1}{\sqrt{r_{v_1} + r_{v_j}}} &= \frac{m-1}{\sqrt{1+1}} \\ &= \frac{m-1}{\sqrt{2}}.\end{aligned} \tag{31}$$

Provided that K_m is symmetric graph, similar finding is gained for the remaining $m-1$ vertices.

Combining the result we get,

$$
\begin{aligned}
&= \frac{1}{\sqrt{2}}\big[(m-1)+(m-2)+(m-3)+\cdots+m-(m-1)\big]\\
&= \frac{1}{\sqrt{2}}\big[(m+m+\cdots+m)(m-1)\text{ times}-(1+2+3+\cdots+m-1)\big]\\
&= \frac{1}{\sqrt{2}}\left[m(m-1)-\frac{m(m-1)}{2}\right]\\
&= \frac{1}{\sqrt{2}}\left[\frac{m(m-1)}{2}\right]. \qquad (32)
\end{aligned}
$$

On adding the equations (30) and (32) we get the required result. □

Theorem 2.7. *The reverse Zagreb indices of families of corona product of graphs* $K_m \circ K'_n$ *are*

$$
\begin{aligned}
RM_1(G) &= mn(m+n-1)^2 + m,\\
RM_2(G) &= mn(m+n-1) + \frac{m(m-1)}{2}.
\end{aligned}
$$

Proof. For the families of corona product of graphs there are mn vertices of the reverse vertex degree $m+n-1$ and m vertices $v_1, v_2, v_3, \ldots, v_m$ of the reverse vertex degree 1, accordingly. Moreover for the vertices $u_1, u_2, \ldots, u_n$, we obtain the following,

$$
\begin{aligned}
\sum_{u_i} r^2_{u_i} = r^2_{u_1} + r^2_{u_2} + \cdots + r^2_{u_n} &= \underbrace{(m+n-1)^2 + (m+n-1)^2 + \cdots + (m+n-1)^2}_{n \ \ \text{times}}\\
&= n(m+n-1)^2. \qquad (33)
\end{aligned}
$$

Since there are mn vertices of reverse vertex degree $(m+n-1)$ and combining the above results for all mn vertices. Thus, the equation (33) becomes,

$$
= \underbrace{n(m+n-1)^2 + n(m+n-1)^2 + \cdots + n(m+n-1)^2}_{m \ \ \text{times}} = mn(m+n-1)^2. \qquad (34)
$$

Also, for the vertex v_1, we have

$$
r^2_{v_1} = 1. \qquad (35)
$$

Since the graph is symmetric, the some result is obtained for remaining $(m-1)$ vertices.

$$
\begin{aligned}
\sum_{v_j} r^2_{v_j} &= r^2_{v_1} + r^2_{v_2} + \cdots + r^2_{v_m}\\
&= \underbrace{1+1+1\cdots+1}_{m \ \ \text{time}}\\
&= m. \qquad (36)
\end{aligned}
$$

On adding equations (34) and (36), we get the required result.

Similarly, one can prove the result for $RM_2(G)$. □

Theorem 2.8. *The reverse arithmetic-geometric and geometric-arithmetic indices of the families of corona product $K_m \circ K_n'$ are*

$$RAG(G) = \frac{mn(m+n)}{2\sqrt{m+n-1}} + \frac{m(m-1)}{2\sqrt{2}},$$

$$RGA(G) = \frac{2mn\sqrt{m+n-1}}{m+n} + \frac{m(m-1)}{\sqrt{2}}.$$

Proof. Theorem 2.6 is analogous to this proof. ☐

Theorem 2.9. *The reverse Sombor index of family of corona product of graph $K_m \circ K_n'$ is*

$$RSO(G) = mn\sqrt{1+(n+m-1)^2} + \frac{m(m-1)}{2}.$$

Proof. Theorem 2.6 is analogous to this proof. ☐

Theorem 2.10. *The reverse Nirmala index of family of corona product of graph $K_m \circ K_n'$ is*

$$RN(G) = mn\sqrt{n+m} + \frac{m(m-1)}{\sqrt{2}}.$$

Proof. Theorem 2.6 is analogous to this proof. ☐

3 Conclusions

Numerous scholarly papers have addressed the computation of topological indices in various families of graphs. While some of these studies have yielded practical applications. Like,

1. **Social Network Analysis:**
   Modeling social networks with n and m representing different entities enables the identification of key individuals and community structures, enhancing our understanding of group dynamics.
2. **Communication Networks:**
   Using the corona product $(K_m \circ K_n')$ for communication networks optimizes resource allocation between devices (m) and services (n'), enhancing network efficiency.
3. **Data Integration:**
   Bistar graphs capture relationships between data sources (n) and attributes (m), aiding in data consolidation and quality assessment.
4. **Image Processing:**
   Applying bistar graphs to image analysis, with n and m representing pixels, assists in feature extraction and object recognition, benefiting computer vision and image processing applications. These findings not only enhance our understanding of the topological properties inherent to these graph families but also hold promise for making substantial contributions to the fields of network science and graph theory. Moreover, these indices may find practical applications in diverse areas such as network analysis and connectivity assessment.

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